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"THE DOMINANT CELL METHOD; OPTIMAL SOLUTION FOR LINEAR TRANSPORTATION PROBLEM"

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ABSTRACT

This research offers a new method to accomplish the optimal solution (O.S) of transportation problem (T.P) in linear programming (L.P). It is simple and direct unlike the current method which requires mostly two stages; one for feasible basic solution (FBS) and the other for developing it in to optimal solution (O.S) by stepping stone Method (SSM) or Modified Distribution Method (MDM). The proposed method is called ‘‘ Dominant cell Method ’’ (DCM).

KEYWORDS: Linear Programming (L.P), Transportation Problem (T.P), Feasible Basic Solution (FBS), Optimal Solution (O.S), Stepping Stone Method (SSM), Modified Distribution Method (MDM) & Dominant Cell Method (DCM)

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Original Article

1. INTRODUCTION

(T.P.) is today, an important problem in (L.P.) for any productive management because the Transportation has become a part of the productive process that determines the price of the produced goods and the profits. The interest of (T.P.) goes back to 1941 when F.L. Hichcok offered a study called"The distribution of production from sources to different points". [3].

Then in 1947, T.C. Koopmans offered his study named. "The optimal use of Transportation system" which is developed by G.B. Dantziginto, the simplex, after wards. But the special nature of (T.P) which permits to find easily feasible basic solution (FBS) made some researchers focus their efforts on finding special methods to develop the (FBS) into (O.S.) like Dantzig and others in 1951 when they offered (MDM), and like Charnes and Cooper in 1954 who offered (SSM). Whereas Other researchers thought to find methods of attaining (FBS) which is also (O.S) in the same time or at least nearest to it like (Vogel) in 1958 and Russel in 1968. [4]. All of them found that indirect way is more efficient to get (O.S) than the simplex, although it involves two or three stages sometimes. [5].

In this paper, I tried to find a simple and direct way for attaining (O.S) Far from the simplex and the special methods for (FBS) which can be developed into (O.S). I proposed a direct and simple method by which one can accomplish (O.S) easily.

I called this proposed methods "Dominant cell Method" (DCM) because I think that there is always at least a (D.C) in each step of solving the balanced (T.P) can be used frequently for transportation till getting (O.S). For this objective, the research is divided into four parts; the first for problem, objective and the assumption. The second for Transportation Matrix (T.M) and the mathematical model, in addition to the concept of (D.C) and

how one can diagnose and use it for (O.S).

The third, for applying (DCM) on some (T.P) and testing its optimal solutions by (MDM). Also I presented the final solutions of the special and common methods for comparison. The last part is devoted to "Conclusions and Suggestions".

2. THE PROBLEM, OBJECTIVE AND ASSUMPTION

2.1 The Problem and the Importance of the Research

First of all the importance of this research is related to the importance of goods transportation from supply to demand points with Least cost. The problem here, is how one can choose the right route (cell) in (T.M) to meet the demand ; that is, how to choose the cell which leads to minimize the transportation cost and then maximize the profit of the productive management. The available solution of this problem, was, the simplex, But the researchers today gave up using the simplex in solving (T.P) when some of them found another way to accomplish (O.S) which is more efficient and requires two stages:

First: Find (FBS) by using one of the common methods [North- west corner (NWCM), least costs (LCM) and Vogel's Approximation (VAM)].

Second: Test the optimization of this (FBS) and develop it in to (O.S) if it is not.

But this way to find the (O.S) requires many steps some times and satisfying the condition of the equality between the number of used cells in (FBS) and the linear constraints of (T.P) which is not available always.

2.2 The Objective

This research tries to find a new method to accomplish the (O.S) of (T.P) directly and simply with a few steps by determining the (D.C) of (T.M) at each step of (O.S) and using it to meet the demand from supply points.

2.3 Assumption of the Research

The assumption of this research can be written in statistical method as following:

Ho: There is no (D.C) at each step of solving (T.P) that can be determined and used to accomplish (O.S) directly and easily.

Hi: There is a (D.C) at each step of solving (T.P) that can be determined and used to accomplish (O.S) directly and easily.

3. THEORITICAL PART

3.1 Mathematical Transportation Model

Let Si (i = 1,..., m) are supply points of known amounts of goods di (i = 1,..., m) to be shipped to demand points Dj (j= 1,...,n) according to known demands bj (j= 1,...,n) with least possible cost where the cost of shipping each unit of goods is also known cij (i=1,...,m ; j= 1,...,n). Now suppose that (T.P) is balanced, i.e:[*]

$$\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj = a = b. \tag{1}$$

[*]It is easy to change the unbalanced (T.P) into balanced by adding to (T.M) either another row or column of cells with zero costs according to the need of balance.

And xij are the shipped goods from the supply point (i) to demand point (j) then:

$$\sum_{j=1}^{n} xij = ai \;\; ; \; i = 1,2, \dots , m \tag{2}$$

$$\sum_{i=1}^{m} xij = bj \;\; ; \; j = 1,2, \dots , n \tag{3}$$

From (1), (2) and (3) the independent linear conditions that represent the basic variables for solving (T.P) equal (m+n-1) and the mathematical model of (T.P) is:

$$\text{Min.}\, Z = \sum_{i=1}^{m} \sum_{j=1}^{n} cijxij$$

Subject to:

$$\sum_{j=1}^{n} xij = ai \;\; ; \; i = 1,2, \dots , m$$

$$\sum_{i=1}^{m} xij = bj \;\; ; \; j = 1,2, \dots , n$$

$$xij \geq 0 \quad \forall \, i , j$$

3.2 Transportation Matrix (T.M) and the (O.S) Problem

According to the assumptions from 1 to 3 in (3-1) the (T.P) can be presented as the following matrix:

Table 1

Di / Si	D_1	D_2	....	Dn	ai
S_1	$\boxed{C_{11}}$ X_{11}	$\boxed{C_{12}}$ X12	....	$\boxed{C_{1n}}$ X_{1n}	a_1
S_2	$\boxed{C_{21}}$ X_{21}	$\boxed{C22}$ X_{22}	....	$\boxed{C_{2n}}$ X_{2n}	a_2
. . . .	. . . .	. . . .	. . . .	. . . .	. . . .
Sm	$\boxed{C_{m1}}$ X_{m1}	$\boxed{C_{m2}}$ X_{m2}	....	$\boxed{C_{mn}}$ X_{mn}	am
Bj	b_1	b_2	....	bn	a / b

From the (T.M) above, xij are just the unknown quantities to be shipped by cells from supply to demand points at least total cost. Since xij and cij are different from cell to another and they are determined by sum of row or column, which of them is less, then the chosen cell is the determinant of the shipment and its cost. Therefore xij are not the key of solving (T.P) but it is the chosen cell itself. This logic leads some researchers to choose cells according to the minimum costs like (LCM) which was advanced step to (NWCM). But the (LCM) mostly still offers (FBS) not (O.S) except when the distribution of costs in (T.M) are such that each row and column contain a minimum cost. Vogel tried to determine the chosen cell depending on two minimum costs instead of one especially those with greatest difference. Although this method accomplished the (O.S) in some (T.P) more than (LCM) but it also failed in many others as we shall see in applied part of this paper because of the distribution of the costs in (T.P). Now how one can choose the right cell to be used for transportation and get the (O.S) directly and easily. I think the following of this paper carry the answer to that big question.

3.3 The Dominant Cell (D.C); Concept, Conditions and How to Use for (O.S)

3.3.1 The Concept of (D.C)

It is the cell of least cost (if possible) in row or column of the greatest range of costs of (T.P) which leads when it is used to omit the row or column (which of them is less) on condition that there will be no effects stand before omitting another row or column of the rest (T.M) by using cells with small costs if possible.

This condition, which will be explained in detail later, is already available when all cells of omitted row / column are not with small costs except the (D.C). It is working only when there is at least one cell of small cost is omitted without using for transportation. Also, when (T.M) is (2×n) where $n \geq 2$ or (m×2) where $m > 2$. In this case there are only two cells in each row or column and the one with less cost can be used according to the rule of greatest range of costs difference.

3.3.2 Conditions of (D.C) in all (T.M) Except (2×n) and (m×2): We have the following Two Cases

3.3.2.1 The Use of (D.C) Leads to Omit its Column

- It is better not to be the only cell of small cost in its row, but if it is, then it should lead to omit the row also or make acute minimization in its summation at least.
- It's cost should not be frequent in other cells of its omitted column.
- The cells of small costs in the omitted column should not be the unique in their rows and it is better to be sufficient to omit their rows and columns.

3.3.2.2 The Use of (D.C) Leads to Omit its Row

- It is better not to be the only cell of small cost in its column but if it is, then it should lead to omit the column also or make acute minimization in it at least.
- Its cost should not be frequent in other cells of its omitted row.
- The cells of small costs in the omitted row should not be the unique in their columns and it is better to be sufficient to omit their columns and rows.
- Its use should not lead to omit a cell in its row which is of least cost in its column.

That is, the conditions for omitting the row by using (D.C) are themselves the conditions for omitting the column after replacing the word "row" by the word "column" and take care not to omit a cell in row which is of least cost in its column.

3.3.3 How to Use (D.C) for (O.S): The Procedures of (DCM)

- Order the costs of (T.P) ascending from left to right and separate between the great costs and small costs by median cost.
- Find the range of costs for only each row and column which contains costs more than and less than median cost.
- Choose the row/ column of greatest range and use the cell of least cost in it if it meets conditions of (D.C) otherwise, go to another small cost cell in the same row/ column and examine it. If all the small cost cells in that

row/column do not meet the conditions of (D.C) then leave it to the same frequent difference if existing, or to the nearest and do the same till you get the (D.C) then use it to omit the row or the column (which is of less summation).

- Repeat the steps (2 and 3) till you meet all the demands.

Note 1: In case of (T.P) with 2×n, $n \geq 2$ or m×2, m>2 the range of costs is between each two cells of only columns in (2×n) and between each two cells of only rows in (m×2), starting descending with cells of greatest range and noticing the summations of rows and columns. So this case of (T.P) can be solved with one step only and without looking for satisfying the conditions of (D.C). Now, in this case, if the greatest range is frequent, then it is better to choose the one with greatest cost to be omitted first.

Note2: If the medium cost is very nearest to one small cost, then we can treat it as a small cost.

4. APPLIED PART

Because of the nature of this research; it is just a paper, I shall apply and explain the procedure of my proposed method (DCM) on only some problems and I shall use (MDM) to test the optimization of the solutions.

Also, I shall offer the final solutions (minimum costs) given by the other special methods (NWCM, LCM, VAM) in order to show the difference between them and my proposed Method.

First Problem

Table 2

	D_1	D_2	D_3	D_4	
s_1	5	7	3	6	10
s_2	3	2	9	9	15
s_3	13	11	10	12	20
	8	12	18	7	

a- The sol: 2, 3, 5, 6, 7, 9, 10, 11, 12, 13

Table 3

	~~D_1~~	D_2	D_3	D_4	
s1	5	7	3	6	10
s2	3 8	2	9	9	~~15~~ 7
s3	13	11	10	12	20
	~~8~~	12	18	7	
	(10)	(9)	(7)	(6)	

upper range in D_1 ⟹ (DC) is S_2 D_1

	D_2	D_3	D_4	
s1	7	3	6	10
~~s2~~	2 7	9	9	~~7~~
s3	11	10	12	20
	5 ~~12~~	18	7	
	(9)	(7)	(6)	

Table 4

U.d. in D_2 $\Longrightarrow$ (DC) is $S_2 D_2$

	D_2	D_3	D_4	
~~s1~~	7	3 (10)	6	~~10~~
s3	11 (5)	10 (8)	12 (7)	20
	5	18	7	
	(4)	↑(7)	(6)	

U.R. in D_3 $\Longrightarrow$ (DC) is $S_1 D_3$

	D_1	D_2	D_3	D_4	
S_1	5	7	3 (10)	6	10
S_2	3 (8)	2 (7)	9	9	15
S_3	13	11 (5)	10 (8)	12 (7)	20
	8	12	18	7	

$$\min. Z = 10(3) + 8(3) + 7(2) + 5(11) + 8(10) + 7(12)$$

$$= 30 + 24 + 14 + 55 + 80 + 84 = 287 \text{ by DCM}$$

$$= 333 \text{ by NWCM}$$

$$= 292 \text{ by LCM}$$

$$= 302 \text{ by VAM}$$

Notices

- m+n-1 = 6 = no. of used cells.
- The rows of the matrix did not contain high costs and low costs so the differences are taken for columns only.
- The last step treats with special matrix (2×n) and n>2 so it is done in one step only.

b- Test of Optimization

(1) For used cells: $ui + vj = cij$

$u_1 + v_3 = 3 \Rightarrow u_1 = 0 \Rightarrow v_3 = 3\,;\; u_2 + v_1 = 3 \Rightarrow v_1 = 5$

$u_2 + v_2 = 2 \Rightarrow u_2 = -2\,;\; u_3 + v_2 = 11 \Rightarrow v_2 = 4$

$u_3 + v_3 = 10 \Rightarrow u_3 = 7\,;\; u_3 + v_4 = 12 \Rightarrow v_4 = 5$

(2) For unused cells: $\bar{c}ij = cij - (u_i + v_j)$

$\bar{c}_{11} = 0\,;\; \bar{c}_{12} = 3\,;\; \bar{c}_{14} = 1$

$\bar{c}_{23} = 8\,;\; \bar{c}_{24} = 7\,;\; \bar{c}_{13} = 1$

Because there is no any negative indirect cost value of the unused cells, then the solution is optimal.

Second Problem

Table 5

	D_1	D_2	D_3	D_4	D_5	
s_1	1	9	13	36	51	50
s_2	24	12	16	20	1	100
s_3	14	33	1	23	26	150
	100	70	50	40	40	

a- The sol: 1, 9, 12, 13,14,16,20,23, 24, 26, 33, 36, 51

Table 6

		D_1	D_2	D_3	D_4	~~D_5~~		
(50)	s_1	1	9	13	36	51	50	
(23)	s_2	24	12	16	20	1 40	~~100~~ 60	U.R. in D_5
(32)	s_3	14	33	1	23	26	150	(DC) is S_2 D_5
		100	70	50	40	~~40~~		
		(23)	(24)		(16)	(50)		

		D_1	D_2	~~D_3~~	D_4	
(35)	s1	1	9	13	36	50
(12)	s2	24	12	16	20	60
(32)	s3	14	33	1 50	23	~~150~~ 100
		100	70	~~50~~	40	
		(23)	(24)			

U.R. in S_3

(DC) is S_3 D_3

		D_1	D_2	D_4		
(35)	s_1	1	9	36	50	
(12)	~~s_2~~	24	12 60	20	~~60~~	U.R. in D_2
(19)	s_3	14	33	23	100	
		100	~~70~~ 10	40		
		(23)	(24)			

	~~D_1~~	~~D_3~~	D_4	
s1	1 40	9 10	36	50
s3	14 60	33	23 40	~~100~~ 60
	~~100~~	~~10~~	40	
		(24)	(13)	

Table 7

U.R. in D_2 $\Longrightarrow$ DC is S_1D_2

	D_1	D_2	D_3	D_4	D_5	
S_1	$\boxed{1}$ 40	$\boxed{9}$ 10	$\boxed{13}$	$\boxed{36}$	$\boxed{51}$	50
S_2	$\boxed{24}$	$\boxed{12}$ 60	$\boxed{16}$	$\boxed{20}$	$\boxed{1}$ 40	100
S_3	$\boxed{14}$ 60	$\boxed{33}$	$\boxed{1}$ 50	$\boxed{23}$ 40	$\boxed{26}$	150
	100	70	50	40	40	

$$\min. Z = 40(1) + 10(9) + 60(12) + 40(1) + 60(14) + 50(1) + 40(23)$$

$$= 40 + 90 + 720 + 40 + 840 + 50 + 920 = 2700 \text{ by DCM}$$

$$= 2810 \text{ by LCM}$$

$$= 3520 \text{ by VAM}$$

$$= 4520 \text{ by WNCM}$$

Notices:

- m+n-1 = 7 = used cells.
- $m + n - 1 \neq 7$ in VAM
- The first D.C. is in D_5 not in S_1 because the small cost cells in S_1 do not satisfy the conditions of D.C.
- The last step treats with special matrix (2×n) so it is done immediately with one step only.

b - Test of Optimization

(1) For used cells: $ui + vj = cij$

$u_1 + v_1 = 1 \Rightarrow u_1 = 0, v_1 = 1\ ;\ u_1 + v_2 = 9 \Rightarrow v_2 = 9$

$u_2 + v_2 = 12 \Rightarrow u_2 = 3\ ;\ u_2 + v_5 = 1 \Rightarrow v_5 = -2$

$u_3 + v_1 = 14 \Rightarrow u_3 = 13\ ;\ u_3 + v_3 = 1 \Rightarrow v_3 = -12$

$u_3 + v_4 = 23 \Rightarrow v_4 = 10$

(2) For unused cells: $\bar{c}ij = cij - (u_i + v_j)$

$\bar{c}_{13} = 25\ ;\ \bar{c}_{14} = 26\ ;\ \bar{c}_{15} = 53$

$\bar{c}_{21} = 20\ ;\ \bar{c}_{23} = 25\ ;\ \bar{c}_{24} = 7$

$\bar{c}_{32} = 11\ ;\ \bar{c}_{35} = 15$

Then the solution is optimal.

Third Problem: [1]

Table 8

	D_1	D_2	D_3	D_4	
S_1	2	3	7	11	150
S_2	0	12	5	6	125
S_3	14	1	3	9	75
S_4	10	2	5	8	50
	100	20	80	200	400

a- sol: 0, 1,2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 14

Table 9

		~~D_1~~	D_2	D_3	D_4	
(9)	S_1	2 (100)	3	7	11	50 ~~150~~
(12)	S_2	0	12	5	6	125
(13)	S_3	14	1	3	9	75
(8)	S_4	10	2	5	8	50
		~~100~~ (14)	20 (11)	80	200 (5)	400

U.R in D_1 ⟹ (DC) is $S_1 D_1$

		~~D_2~~	D_3	D_4	
(8)	S_1	3 (20)	7	11	~~50~~ 30
(7)	S_2	12	5	6	125
(8)	S_3	1	3	9	75
(6)	S_4	2	5	8	50
		~~20~~ (11)	80	200 (5)	

Table 10

U.R. in D_2 ⟹ (DC) is $S_1 D_2$

		D_3	D_4	
(4)	S_1	7 (5)	11 (25)	~~30~~
	S_2	5	6 (125)	~~125~~
(6)	S_3	3 (75)	9	~~75~~
(3)	S_4	5	8 (50)	~~50~~
		~~80~~	~~200~~	

U.R. in S_3 ⟹ (DC) is $S_3 D_3$

	D_1	D_2	D_3	D_4	
S_1	2 (100)	3 (20)	7 (5)	11 (25)	150
S_2	0	12	5	6 (125)	125
S_3	14	1	3 (75)	9	75
S_4	16	2	5	8 (50)	50
	100	20	80	200	

$$\min. Z = 100(2) + 20(3) + 5(7) + 25(11) + 125(6) + 75(3) + 50(8)$$

$$= 200 + 60 + 35 + 275 + 750 + 225 + 200 = 1945 \text{ by (DCM)}$$

$$= 2245 \text{ by (VAM)}$$

$$= 2245 \text{ by (NWCM)}$$

$$= 2310 \text{ by (LCM)}$$

Notices:

- m+n-1 = 7 = no. of used cells.
- The first D.C is in D_1 ; It is S_1 D_2 not S_2 D_1 and the second D.C. in D_2 is S_1 D_2 neither S_3 D_1 nor S_4 D_1.
- The third step represents a special matrix it is (m×2) and m>2; so it is already done in one step by (DCM).

b- Test of Optimization

(1) For used cells: $ui + vj = cij$

$u_1 + v_1 = 2$; $u_1 = 0$ &$v_1 = 2$; $u_1 + v_2 = 3$; $v_2 = 3$; $u_1 + v_3 = 7 \Rightarrow v_3 = 7$

$u_1 + v_4 = 11 \Rightarrow v_4 = 11$; $u_2 + v_4 = 6 \Rightarrow u_2 = -5$; $u_3 + v_3 = 3 \Rightarrow u_3 = -4$

$u_4 + v_4 = 8 \Rightarrow u_4 = -3$

(2) unused cells: $\bar{c}ij = cij - (u_i + v_j)$

$\bar{c}_{21} = 3$; $\bar{c}_{22} = 14$; $\bar{c}_{23} = 3$; $\bar{c}_{31} = 16$; $\bar{c}_{32} = 2$; $\bar{c}_{34} = 2$

$\bar{c}_{41} = 11$; $\bar{c}_{42} = 2$; $\bar{c}_{43} = 1$

So the solution is optimal.

Fourth Problem: [2]

Table 11

	D_1	D_2	D_3	D_4	
S_1	20	22	17	4	120
S_2	24	37	9	7	70
S_3	32	37	20	15	50
	60	40	30	110	

a- sol: 4, 7, 9, 15, 17, 20, 22, 24, 32, 37

Table 12

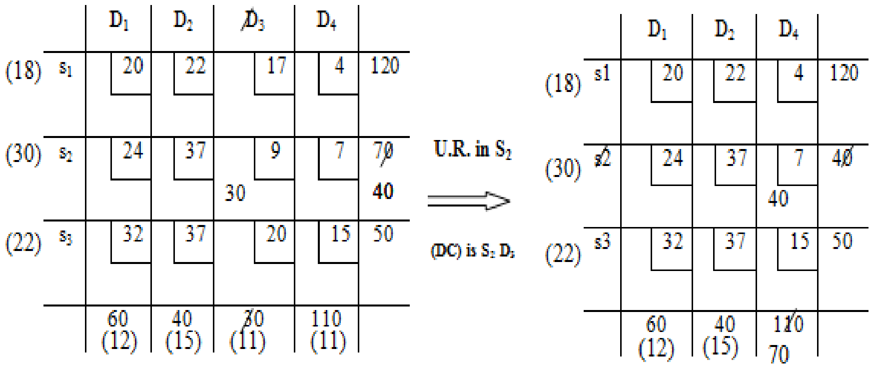
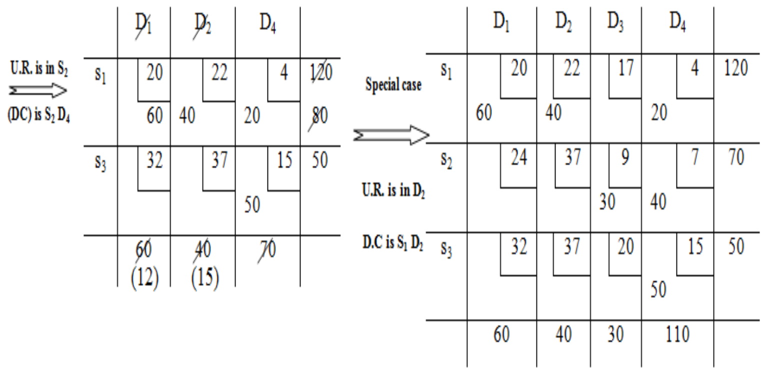


Table 13



$$\min. Z = 60(20) + 40(22) + 20(4) + 9(30) + 7(40) + 15(50)$$

$$= 1200 + 880 + 80 + 270 + 280 + 750 = 3460 \text{ by (DCM)}$$

$$= 3520 \text{ by (VAM)}$$

$$= 3670 \text{ by (LCM)}$$

$$= 3680 \text{ by (NWCM)}$$

Notices

- m+n-1 = 6 = no. of used cells.
- The first D.C is S_2 D_3 not S_2 D_4.
- In third step we get a special case; a matrix of (2×n) & n>2 Then it is already done in one step only by (DCM).

b- Test of Optimization

(1) For used cells: $ui + vj = cij$

$$u_1 + v_1 = 20 \Rightarrow u_1 = 0 \Rightarrow v_1 = 20\,;\ v_2 = 22\,; v_4 = 4, v_3 = 6\ u_2 = 3, u_3 = 11$$

(2) For unused cells: $\bar{c}ij = cij - (u_i + v_j)$

$\bar{c}_{13} = 11\,;\bar{c}_{21} = 1\,;\ \bar{c}_{22} = 12$

$\bar{c}_{31} = 1\,;\bar{c}_{32} = 4, \bar{c}_{33} = 3$

So the solution is optimal.

Fifth Problem: [6]

Table 14

		D_1	D_2	~~D_3~~	~~D_4~~	D_5	
(10)	s_1	37	27	28 70	34	30	~~100~~ 30
(5)	s_2	29	32	32	27	28	125
(10)	s_3	34	27	37	30	30	150
		75 (8)	60 (5)	~~70~~ (9)	~~80~~ (7)	~~90~~	375

U.R in S_1 ⟹ (DC) is $S_1\,D_3$

a- sol: 27, 28, 29, 30, 32, 34, 37

Table 16

		D_1	D_2	~~D_4~~	D_5	
(10)	~~s_1~~	37	27 30	34	30	~~30~~
(5)	s_2	29	32	27 30	28	125
(7)	s_3	34	27	30	30	150
		75 (8)	60 (5)	~~80~~ (7)	90	

U.R. in S_1 ⟹ (D.C) is $S_1\,D_2$

	~~D_1~~	~~D_2~~	D_3	D_4	
s_2	29 75	32	27 50	28	~~125~~ ~~50~~
s_3	34	27 30	30 30	30 90	~~150~~ ~~120~~ ~~20~~
	~~75~~ (5)	~~30~~ (5)	80 (3)	90 (2)	

Special case ⟹ Matrix of 2×n

	D_1	D_2	D_3	D_4	D_5	
s_1	37	27 30	28 70	34	30	100
s_2	29 75	32	32	27 50	28	125
s_3	34	27 30	37	30 30	30 90	150
	75	60	70	80	90	

$$\min. Z = 30(27) + 70(28) + 75(29) + 50(27) + 30(27) + 30(30) + 90(30)$$

$= 810 + 1960 + 21751 + 1350 + 810 + 900 + 2700 = 10705$ by (DCM)

$= 10935$ by (VAM)

$= 11170$ by (LCM)

$= 11850$ by (NWCM)

Notices

- m+n-1 = 7 = no. of used cells.
- The first D.C is neither S_1 D_2 nor S_3 D_2, it is S_1D_3.
- At the last step we get a special matrix (2×n) then the minimum cost is already done in one step.

b- Test of optimization:

(1) For used cells: $ui + vj = cij$

$u_1 + v_2 = 27 \Rightarrow u_1 = 0$ &$v_2 = 27$; $v_3 = 28$; $u_2 + v_1 = 29 \Rightarrow v_1 = 32$

$u_2 + v_4 = 27 \Rightarrow u_2 = -3$; $u_3 + v_2 = 27 \Rightarrow u_3 = 0$;

$u_3 + v_4 = 30 \Rightarrow v_4 = 30$; $u_3 + v_5 = 30 \Rightarrow v_5 = 30$

(2) For unused cells: $\bar{c}ij = cij - u_i - v_j$

$\bar{c}_{11} = 5, \bar{c}_{14} = 4$; $\bar{c}_{15} = 0$; $\bar{c}_{22} = 8$; $\bar{c}_{23} = 7$; $\bar{c}_{25} = 1$; $\bar{c}_{31} = 2, \bar{c}_{33} = 9$

Then the solution is optimal.

5. CONCLUSIONS & SUGGESTIONS

Conclusions

- There is a "Dominant cell" (D.C) at each step of the optimal solution (O.S) steps of Transportation problem (T.P) and it can be chosen by using (DCM).
- (DCM) offers (O.S) of (T.P) directly with less, simple and frequent steps.
- (DCM) offers (O.S) of (T.P) when it is of 2 × n or m ×2matrix in only one step.
- (DCM) offers (O.S) that mostly satisfies the linear conditions of (T.P) where the no. of used cells equals (m + n – 1).

Suggestions

I suggest (DCM) as easy, direct and quick method for (O.S) of (T.P) provided that the user can always determine the (D.C). Even though he/she can't make the right choice of cell at any step of solution, the method will still offer a special feasible solution but not optimum.

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